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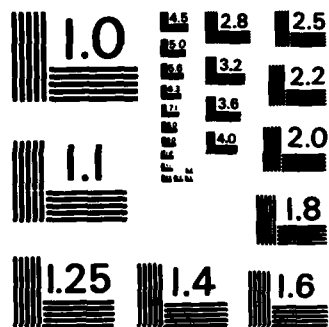
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AN APPLICATION OF THE PERRON-FROBENIUS
THEOREM TO A DAMAGE MODEL PROBLEM*

by

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King Saud University, Saudi Arabia,
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Summary Using the Perron-Frobenius theorem, it is established that if (X, Y) is a random vector of non-negative integer valued components such that $Y \leq X$ almost surely and two modified Rao-Rubin conditions hold, then under some mild assumptions the distribution of (X, Y) is uniquely determined by the conditional distribution of Y given X . This result extends the recent unpublished work of Shanbhag and Taillie (1979) on damage models. *keywords*

Key Words: Damage models; Modified Rao-Rubin condition; Perron-Frobenius theorem.

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1. INTRODUCTION

Let (X, Y) be a random vector with non-negative integer valued components such that

$$P(X=x, Y=y) = g_x S(y|x), \quad y=0,1,\dots,x; \quad x=0,1,\dots \quad (1.1)$$

where $g_x = P(X=x)$, $x=0,1,\dots$ is the probability distribution of X and $S(y|x) = P(Y=y|X=x)$, $y=0,1,\dots,x$, is the conditional probability distribution of Y given $X=x$. Rao and Rubin (1964), Shanbhag (1977) and several others have identified, under certain conditions, either the class of distributions $\{g_x\}$ or that of the conditional distributions $\{S(y|x)\}$ for which the following assertion is valid

$$P(Y=y) = P(Y=y|X=Y), \quad y=0,1,\dots \quad (1.2)$$

which is known as the Rao-Rubin condition. In particular, if $g_0 < 1$ and

$$S(y|x) = \binom{x}{y} \pi^y (1-\pi)^{x-y}, \quad y=0,1,\dots,x; \quad x=0,1,\dots \quad (1.3)$$

then, Rao-Rubin (1964) showed that $\{g_x\}$ is a Poisson distribution. Shanbhag (1977) established the more general result that if $g_0 < 1$ and

$$S(y|x) \propto a_y b_{n-y}, \quad y=0,1,\dots,x \quad (1.4)$$

for each x for some positive sequence $\{a_n\}$ and non-negative sequence $\{b_n\}$ with $b_0, b_1 > 0$, then (1.2) is satisfied if and only if

$$g_x \propto c_x \lambda^x, \quad x=0,1,\dots$$

for some λ , where $\{c_n\}$ is the convolution of $\{a_n\}$ and $\{b_n\}$. Slightly more general

versions of this result are given by Shanbhag (1983) and Rao and Lau (1982).

A natural question that arises is whether the same results can be obtained by modifying the condition (1.2) to

$$P(Y=y) = P(Y=y|X-Y=k), \quad y=0,1,\dots \quad (1.5)$$

for any fixed $k > 0$ such that $P(X-Y=k) \neq 0$, which will be referred to as the $RR(k)$ condition. [In this notation, the original condition (1.2) will be $RR(0)$.]

Srivastava and Singh (1975) conjectured that the Rao-Rubin (1964) result will hold even under an $RR(k)$ condition with $k > 0$. Patil and Taillie (1979) have shown that the conjecture is false by constructing a counter example. However they have shown that the result is valid under the damage model (1.3) if $RR(k)$ holds for at least two values of k . Using the more general damage model (1.4) and arguments similar to those in Shanbhag (1977), Shanbhag and Taillie (1979, unpublished note) characterized the distribution of X under two $RR(k)$ conditions. (See also Alzaid (1983) and Alzaid et al (1985b) for further remarks on this conjecture).

In this paper, we establish a general characterization theorem under two $RR(k)$ conditions using the Perron-Frobenius theorem concerning primitive matrices given in Seneta (1973, pp. 1-6). This provides a new proof and an extension of the unpublished result of Shanbhag and Taillie (1979).

2. THE MAIN THEOREM

Let $\{g_x\}$: $x=0,1,\dots$ and for each $x \geq 0$, $\{S(y|x)$: $y=0,1,\dots,x\}$ be probability distributions, and $k_0 \geq 0$ and $k_1 > 0$ be fixed integers. Define

$$S_{ij}^* = \sup\{S(j+mk_1|i+k_0+nk_1): m \geq 0, n \geq 0, k_0 \leq j+mk_1 \leq i+k_0+nk_1\}.$$

THEOREM Let the random vector (X, Y) be such that

$$P(X = x, Y = y) = g_x S(y|x), \quad y = 0, 1, \dots, x; \quad x = 0, 1, \dots$$

where g_x and $S(y|x)$ satisfy the following conditions:

- (i) $g_x > 0$ for some $x \geq k_0 + k_1$.
- (ii) $S(x + rk_1 | x + k_0 + (r+1)k_1) > 0$ for each $x = 0, 1, \dots, k_1 - 1$, $i = 0, 1$ and $r = 0, 1, \dots$, and $S(x|x) > 0$ for each $x = 0, 1, \dots, k_0 - 1$ (when $k_0 > 0$).
- (iii) The matrix $S^* = (S_{ij}^*)$, $i, j = 0, 1, \dots, k_1 - 1$, is irreducible.

Then, under the two RR(k) conditions

$$P(Y = y) = P(Y = y | X - Y = k_0) = P(Y = y | X - Y = k_0 + k_1), \quad y = 0, 1, \dots, \quad (2.1)$$

and for a fixed value $\lambda = P(X - Y = k_0 + k_1) / P(X - Y = k_0)$, the family of conditional distributions

$$\{S(y|x): y = 0, 1, \dots, x; x = 0, 1, \dots\}$$

determines the $\{g_x: x = 0, 1, \dots\}$ uniquely and $g_x > 0$ for $x \geq k_0$.

Proof Using the equation $P(Y = y | X - Y = k_0) = P(Y = y | X - Y = k_0 + k_1)$ of the condition (2.1) and the assumption (ii) of the theorem we have

$$g_{y+bk_1+k_0} = \lambda g_{y+(b-1)k_1+k_0} \frac{S(y+(b-1)k_1 | y+(b-1)k_1+k_0)}{S(y+(b-1)k_1 | y+bk_1+k_0)}$$

$$y = 0, 1, \dots, k_1 - 1, \quad b = 1, 2, \dots$$

Hence

$$g_{y+bk_1+k_0} = \lambda^b g_{y+k_0} \prod_{m=0}^{b-1} \frac{S(y+mk_1 | y+k_0+mk_1)}{S(y+mk_1 | y+k_0+(m+1)k_1)}$$

$$y = 0, 1, \dots, k_1 - 1, \quad b = 1, 2, \dots \quad (2.2)$$

which, in view of the assumption (i), implies that

$$g_{k_0} + \dots + g_{k_0+k_1-1} \neq 0.$$

The system of equations $P(Y=y) = P(Y=y|X-Y=k_0)$, $y = 0, 1, \dots$ implies that

$$\frac{g_{y+mk_1+k_0}}{P(X-Y=k_0)} S(y+mk_1|y+mk_1+k_0) = \sum_{x=y+mk_1}^{\infty} g_x S(y+mk_1|x), \quad y, m \geq 0. \quad (2.3)$$

Summing (2.3) over m such that $y+mk_1 \geq k_0$ and using (2.2), we find

$$\lambda^* g_{y+k_0} = \sum_{x=0}^{k_1-1} g_{x+k_0} q_{xy}, \quad y = 0, 1, \dots, k_1-1 \quad (2.4)$$

where $\lambda^* = [P(X-Y = k_0)]^{-1}$ and q_{xy} are certain uniquely determined functions of $\{S(x|y)\}$. The irreducibility of S^* (assumption (iii)) implies that the matrix (q_{xy}) arrived at in (2.4) is irreducible. The fact that $S(y+mk_1|y+mk_1+k_0) > 0$ for all $m \geq 0$ and $y = 0, 1, \dots, k_1-1$ implies that $q_{xx} > 0$ in (2.4) for all $x = 0, 1, \dots, k_1-1$. Consequently, the matrix $Q = (q_{xy})$ is primitive. Then from the Perron-Frobenius Theorem (Seneta (1973), pp. 1-2) it follows that λ^* is the unique eigenvalue of Q having the largest absolute value and

$$(g_{k_0}, \dots, g_{k_0+k_1-1})$$

is the eigenvector of Q associated with λ^* having all its components positive.

Using (2.2) and (2.3) and the fact that $S(x|x) > 0$ for $x < k_0$, we can express

g_{k_0-1}, \dots, g_0 as linear combinations of $g_{k_0}, \dots, g_{k_0+k_1-1}$ with coefficients which are uniquely determined by $\{S(y|x)\}$. Using the fact that $\sum_{x=0}^{\infty} g_x = 1$ or that $P(X-Y = k_0) = (\lambda^*)^{-1}$, we see that $\{g_x\}$ is uniquely determined by

$\{S(y|x): y = 0, 1, \dots, x; x \geq 0\}$. The remainder of the theorem easily follows.

Remark 1. With appropriate modifications in the assumptions (ii) and (iii) of the theorem, such as replacing $S^* = (S_{ij}^*)$ in (iii) by $S^{**} = (S_{ij}^{**})$ where

$$S_{ij}^{**} = \sup\{S(j+mk_1 | i+k_0+nk_1) : m \geq 0, n \geq 0, k_0 \leq j+mk_1 \leq \min(i+k_0+nk_1, k_0+k_1-1)\}$$

it is possible to prove the validity of the above theorem with (2.1) replaced by a somewhat weaker assumption of the type

$$P(Y=y | X-Y=k_0) = P(Y=y | X-Y=k_1), \quad y = 0, 1, \dots$$

$$P(Y=y) = P(Y=y | X-Y=k_0), \quad y = 0, 1, \dots, k_0+k_1-1. \quad (2.5)$$

Several other possibilities exist.

Remark 2. If the matrix S^* is such that $S_{i,i+1}^* > 0$, $i = 0, 1, \dots, k_1-2$, and $S_{k_1-1,0}^* > 0$, then clearly it is irreducible. Also, if it is such that $S_{i,i-1}^* > 0$, $i = 1, \dots, k_1-1$ and $S_{0,k_1-1}^* > 0$, then it is irreducible. Using this information one could give a slightly weaker but at the same time simpler versions of the theorem.

3. SHANBHAG-TAILLIE RESULT

The following corollary of our main theorem of Section 2 is indeed the Shanbhag Taillie (1979) result.

Corollary Let $\{(a_x, b_x) : x = 0, 1, \dots\}$ be a sequence of vectors with non-negative real components such that $a_x > 0$ for all x and $b_0 > 0$. Let (X, Y) be a random vector with non-negative real components such that for each x with $P(X=x) > 0$, we have

$$P(Y=y | X=x) = a_y b_{x-y} / c_x, \quad y = 0, 1, \dots, x$$

where $\{c_x\}$ is the convolution of $\{a_x\}$ and $\{b_x\}$. Assume that $P(X-Y=k_0) > 0$ and $P(X-Y=k_0+k_1) > 0$ with k_0, k_1 as considered earlier. Then the following conditions are equivalent:

- (i) Y and $X-Y$ are independent.
- (ii) $P(Y=y) = P(Y=y|X-Y=k_0) = P(Y=y|X-Y=k_0+k_1)$, $y=0,1,\dots$
- (iii) For some $\theta > 0$ and some periodic sequence $\{q_x: x=0,1,\dots\}$ with the largest common divisor (l.c.d.) of the x for which $b_x > 0$ as one of its periods,

$$P(X=x) = q_x c_x \theta^x, \quad x=0,1,2,\dots$$

Proof We prove the corollary by showing (i) \Rightarrow (ii) \Rightarrow (iii) \Rightarrow (i). Except for the implication (ii) \Rightarrow (iii), all the others are easy to establish. To show (ii) \Rightarrow (iii), let us assume that (ii) is valid. Let τ denote the l.c.d. of those x for which $b_x > 0$. Define $\{(X_i^*, Y_i^*): i=0,1,\dots,\tau-1\}$ to be a sequence of random vectors such that the joint distribution of X_i^* and Y_i^* is the same as the conditional distribution of $(X-i)/\tau$ and $(Y-i)/\tau$ given that $X \in \{i, i+\tau, i+2\tau, \dots\}$ for each i . Clearly, assuming without loss of generality $P(X \in \{i, i+\tau, i+2\tau, \dots\}) > 0$, for each $i=0,1,\dots,\tau-1$, it follows that for each $i \in \{0,1,\dots,\tau-1\}$

$$P(X_i^* = x, Y_i^* = y) = g_x^*(i) a_y^*(i) b_{x-y}^*(i) / c_x^*(i), \quad y=0,1,\dots,x; \quad x=0,1,\dots$$

with $a_r^*(i) = a_{i+r\tau}$, $b_r^*(i) = b_{r\tau}$, $r=0,1,\dots$, and $\{c_r^*(i)\}$ as the convolution of $\{a_r^*(i)\}$ and $\{b_r^*(i)\}$, and

$$g_x^*(i) \propto P(X=i+\tau x) c_x^*(i) / c_{i+\tau x}^*, \quad x=0,1,\dots$$

Observe that (ii) is valid also for X_i^* and Y_i^* with k_0 and k_1 replaced by k_0/τ and k_1/τ respectively. The required result follows if it is established that for some $\theta_1 > 0$

$$g_x^*(i) \propto c_x^*(i) \theta_1^x, \quad x=0,1,\dots$$

since the form of $\{g_x^*(i)\}$ implies in view of (ii) that for some y

$$\begin{aligned}\theta_i &= \left[\frac{b_{k_0} P(X_1^* - Y_1^* = (k_0 + k_1)/\tau, Y_1^* = y)}{b_{k_0 + k_1} P(X_1^* - Y_1^* = k_0/\tau, Y_1^* = y)} \right]^{\tau/k_1} \\ &= \left[\frac{b_{k_0} P(X - Y = k_0 + k_1, Y = i + y\tau)}{b_{k_0 + k_1} P(X - Y = k_0, Y = i + y\tau)} \right]^{\tau/k_1} \\ &= \left[\frac{b_{k_0} P(X - Y = k_0 + k_1)}{b_{k_0 + k_1} P(X - Y = k_0)} \right]^{\tau/k_1}, \quad i = 0, 1, \dots, \tau - 1\end{aligned}$$

which is clearly independent of i . Consequently, the required result follows if it is established when $\tau = 1$; this is so because (X_1^*, Y_1^*) satisfies the requirement of (X, Y) with $\tau = 1$. Suppose

$$\lambda = P(X - Y = k_0 + k_1) / P(X - Y = k_0).$$

Clearly in that case, the positivity of g_x for $x \geq k_0$ together with the straightforward relation (2.2) appearing in the proof of the main theorem implies that

$$\sum_{x=0}^{\infty} c_x(\lambda^*)^{x/k_1} < \infty$$

where $\lambda^* = \lambda b_{k_0} / b_{k_0 + k_1}$.

The distribution

$$g_x = c_x(\lambda^*)^{x/k_1}, \quad x = 0, 1, \dots \quad (3.1)$$

together with the distributions $\{S(y|x)\}$ satisfies the requirement of the theorem and gives $P(X - Y = k_0 + k_1) / P(X - Y = k_0) = \lambda$. The unique determination of $\{P(X = x)\}$ here implies that it has to be given by $\{g_x\}$ of (3.1). Consequently, we can conclude that (iii)

is valid when $\tau = 1$ and hence for all τ .

Remark 3. In view of the result of Shanbhag (1983), the above corollary remains valid with (iv) given below included in its statement.

$$(iv) \quad P(Y=y) = P(Y=y|X=Y), \quad y=0,1,\dots$$

Remark 4. The $RR(0)$ condition may not specify $\{g_x\}$ uniquely for any specified set $\{S(y|x)\}$ even if all its members are positive unless some conditions are satisfied. One could give several examples to illustrate this point in view of the Poisson-Martin representation in the potential theory of Markoff chains. One such example is as follows.

Example 1. Let $0 < \pi < \pi' < 1$ and $c > 1$ be fixed. It is shown in Alzaid et al (1985a) that there exist infinitely many distributions $\{g_x^*\}$ with probability generating functions G^* such that

$$G^*(1-\pi+\pi t) = c G^*(\pi' t), \quad |t| \leq 1.$$

Define

$$S(y|x) = \begin{cases} \beta_x^{(x)} \pi^y (1-\pi)^{x-y}, & y=0, \dots, x-1, \quad x \geq 1 \\ \beta_x [(\pi')^x - c^{-1} \pi^x], & y=x, \quad x \geq 0 \end{cases}$$

where $\beta_x = c/[c(1-\pi^x) + c(\pi')^x - \pi^x]$, $x \geq 0$ and

$$g_x \propto \beta_x^{-1} g_x^*, \quad x=0,1,\dots$$

If we take $\{g_x\}$ and $\{S(y|x): y=0,1,\dots,x; x=0,1,\dots\}$ as above, then $P(Y=y) = P(Y=y|X=Y=0)$, $y=0,1,\dots$, with $P(X=Y=0) = 1/(1+c)$ which is fixed.

Remark 5. If $\{S(y|x)\}$ is as given in the corollary and the l.c.d. of x for which $b_x > 0$ is unity then, according to the corollary, the class of distributions $\{g_x\}$

corresponding to which two $RR(k)$'s hold is a certain power series family. However, if we take $\{S(y|x)\}$ not in the form given in the corollary, then it does not follow even when all $S(x|x)$ are positive for $(0 \leq x < \infty)$ that the class of $\{g_x\}$ corresponding to which two $RR(k)$'s hold is a power series family. The following is an illustration.

Example 2. Take

$$S(y|x) = (a_y b_{x-y})/c_x, \quad y = 0, 1, \dots, x; \quad x \geq 2,$$

$$S(1|1) = S(0|0) = 1,$$

where $\{a_r\}$ and $\{b_r\}$ are positive sequences with $\{c_r\}$ as their convolution. We choose $\{a_r\}$ and $\{b_r\}$ such that $\sum_0^\infty c_r \lambda^r < \infty$ for all $\lambda \in [0, \alpha)$ for some $\alpha \leq \infty$. Define

$$g_x^{(\lambda)} = \begin{cases} c_x \lambda^x / \sum_{m=0}^\infty c_m \lambda^m, & x \geq 2 \\ a_1 b_0 \lambda / \sum_{m=0}^\infty c_m \lambda^m, & x = 1 \\ (a_0 b_0 + a_0 b_1 \lambda) / \sum_{m=0}^\infty c_m \lambda^m, & x = 0. \end{cases}$$

Observe that with $\{S(y|x)\}$ as defined above, every $\{g_x^{(\lambda)}\}$ produces a vector $(X^{(\lambda)}, Y^{(\lambda)})$ satisfying $RR(k)$ for all $k \geq 2$. However, $\{g_x^{(\lambda)}\}$ here is not a power series family.

4. REFERENCES

- [1] Alzaid, A.A. (1983). Some contributions to characterization theory. Unpublished Ph.D. Thesis, University of Sheffield.
- [2] Alzaid, A.A., Rao, C. Radhakrishna and Shanbhag, D.N. (1985a). An extension of Spitzer's representation theorem with applications. Tech. Rept. 85-11, Univ. of Pittsburgh.

- [3] Alzaid, A.A., Rao, C. Radhakrishna and Shanbhag, D.N. (1985b). Further extensions of Deny's theorem with applications to characterization problems. Tech. Rept. 85-14. Univ. of Pittsburgh.
- [4] Lau, Ka-Sing and Rao, C. Radhakrishna. (1982). Integrated Cauchy functional equation and characterizations of the exponential law. Sankhya A, 44, 72-90.
- [5] Patil, G.P. and Taillie, C. (1979). On a version of the Rao-Rubin theorem. Sankhya A 41, 129-132.
- [6] Rao, C. Radhakrishna and Rubin, H. (1964). On a characterization of the Poisson distribution. Sankhya A 26, 295-298.
- [7] Seneta, E. (1973). Non-negative Matrices. George Allen and Unwin Ltd., London.
- [8] Shanbhag, D.N. (1977). An extension of the Rao-Rubin characterization of the Poisson distribution. J. Appl. Prob., 14, 640-646.
- [9] Shanbhag, D.N. (1983). Review No. 62038 on C.R. Rao and R.C. Srivastava (1979), Mathematical Reviews Vol. 83a, p. 287.
- [10] Shanbhag, D.N. and Taillie, C. (1979). Unpublished.
- [11] Srivastava, R.C. and Singh, J. (1975). On characterizations of the binomial and Poisson distributions based on a damage model. Statistical Distribution in Scientific Work 3 (Eds. Patil, G.P., Kotz, S. and Ord, J.K.) D. Reidel Publishing Company.

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